All about curls and divs

A gentle introduction for those who missed out on vector field theory as undergraduates, which should help dispel the mystery surrounding vector field equations.

'JOULES WATT'

In an earlier discussion[1] I covered a little of James Clerk Maxwell's remarkable work on electric and magnetic fields and how he related his model to the propagation of light. After reading it, a student friend soon took me to task and said that although he had learned that James himself had conjured up the terms *curl* and convergence (nowadays oppositely directed as *divergence*), he still couldn't see the wood for the trees. "You see", he went on, "I'm none the wiser about what the curl and div – to say nothing of *grad* – really mean" In answer I said, "It is all to do with vector and scalar fields; they are the basis." "Oh no!" he replied, "we had a ghastly maths course about them. That course is still a poor one, you know." I knew this observation could be very true, as maths teaching is now somewhat grim in our educational system. Nobody seems to care enough about it.

But what of curl and so on? You do need some knowledge of differential and integral calculus, but most O-level syllabuses contain a little about these topics now, so it isn't too frightening. The only little bit extra you need is a nodding aquaintance with partial differential coefficients[2].

SCALARS AND VECTORS

Nearly everyone knows the difference between a *scalar* quantity and a directed number, or *vector*. You would get some raised eyebrows if in the grocers someone asked for, "A pound of tomatoes, due North please." Or if elsewhere you heard, "I drove my car at 60 mile/h – quite exhilarating." To which a comment was, "Where to?" and you heard the reply, "Oh, anywhere, I just close my eyes and go – it's just the speed that matters!"

Imagine we have entered a region of space; a room, a pond or river, a box in a laboratory – anywhere, to make measurements on some quantity permeating the region. Typically, your measurements might apply to a draught in a room, or the temperature distribution, or the water-flow pattern in the river, or again, the electromagnetic radiation in the box. We call any such region a *field*. It might be small, like the box, bounded by walls of some sort. It could be vast with undefined boundaries, or "go off to infinity" as a mathematician might say.

As an example, consider measuring the temperature in the room. Point by point we record the thermometer reading. Figure 1 shows what might be happening. Eventually the data would apply to the whole volume. We don't say at some position that it is "22°C East of North," or any such thing. The temperature is a *scalar* quality and the whole distribution of our measuring points throughout the volume is the appropriate *scalar field* for this measurement.

The points are distanced apart and we interpolate in between, so that we imagine the field smoothly varying around the region. In fact, the same conditions of 'continuity' and 'differentiability' that interest mathematicians regarding other functions apply here also.

Light a candle. With it we can now investigate the cold draught cutting across our feet. The candle flame bends over pointing to where the draught is going. We judge how strong the draught is by noting the guttering of the flame. So as we crawl about the room, we end up with some idea of how strong the draughts are and the directions in which they are blowing. Plotting all this out, point by point yields the *vector field* of the draught distribution, as Fig.2 shows.

You can think of this type of field as a room full of lines, some crammed together indicating high intensity regions, others widely spaced in the weaker regions. They all stream along in the various directions of 'flow' that meander from point to point. These imaginary lines are the *stream lines* of flux in such dynamic vector fields as fluid flow systems. Victorian river and estuary explorers had a fine old time plotting sources and flows. Modern wind-tunnel technologists engage in the same practices.

James Clerk Maxwell appreciated Michael Faraday's genius in visualizing the 'field lines' in the regions around electrically

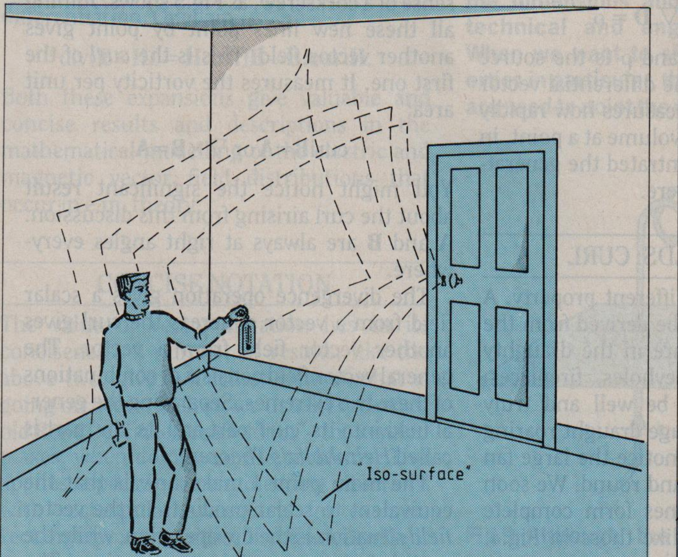

Fig.1. ...invisible surfaces all over your room.

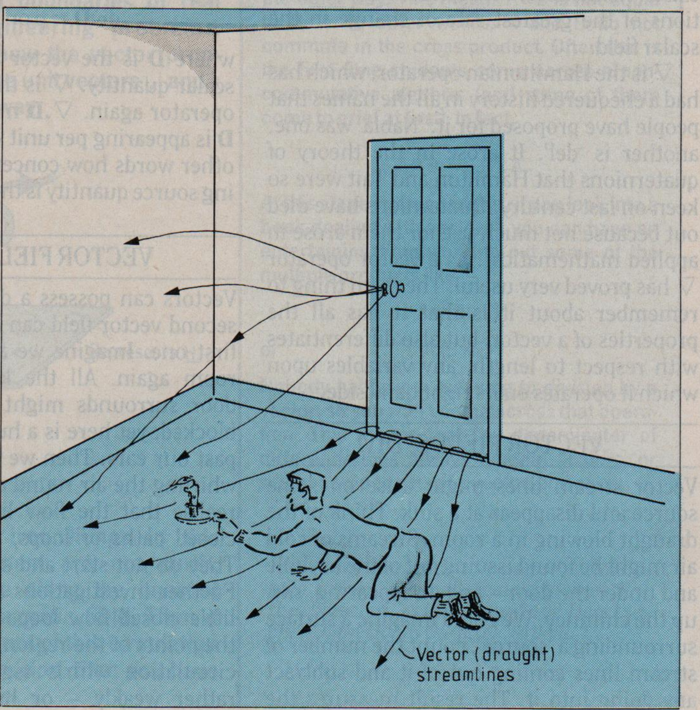

Fig.2. ...crawling about with a vector field roaring about your ears.

charged bodies and around magnets. And as I discussed earlier[1], the whole edifice of the hydrodynamicists with their 'sources and sinks', 'fluxes', 'pressure gradients' (or 'forces'), 'stream functions' and so on, arrived on the electromagnetic field scene with very few changes in terminology. We still have sources and sinks, and fluxes – although no actual flows of electric or magnetic field occur. The one real "flux" situation in our subject is the vector field of conduction current flow.

GRADIENT

If we return to measure the (scalar) temperature distribution all over our imaginary room, we soon discover sets of points at which the thermometer reads the same temperature. We find these points lie on a 'surface' which we might therefore call an *isothermal surface*, which Fig.1 also shows. The iso-surfaces cannot intersect anywhere, or we would have the impossible situation of two different 'constant' values at the same point.

These constant surfaces characterize scalar fields. Another observation soon shows that the temperature changes most rapidly when we move off the isotherm at right angles to it. We could quickly plot all the 'streamlines' of greatest rate of change of temperature, and find they all cross the iso-surfaces at right angles. A vector field results from all this plotting. We have found the *gradient* of the scalar field. We write it as

$$\mathrm{grad}\varphi = \mathbf{A}$$

Alternatively as

$$\nabla\varphi = \mathbf{A}.$$

Here φ is the scalar field *point function.* In other words, it is the magnitude of the field qualtity measured point by point. **A** is the derived vector field distribution point by point. As you now see, we visualize **A** as imaginary stream lines pervading the entire region where the gradient of the scalar field exists. Remember, they arise from the directions of the greatest rate of change in the scalar field.

∇ is the Hamiltonian operator, which has had a chequered history in all the names that people have proposed for it. 'Nabla' was one, another is 'del'. It arose in the theory of quaternions that Hamilton and Tait were so keen on last century. Quaternions have died out because not much use for them arose in applied mathematics[3]. As a vector operator ∇ has proved very useful. The main thing to remember about it is that it has all the properties of a vector, but also differentiates with respect to length, any variables upon which it operates on its right hand side.

VECTOR FIELDS: DIV

Vector stream lines might arise on some source and disappear at a sink. Think of the draught blowing in a room. Streams of cold air might be found issuing out of the keyhole and under the door – and disappearing, say, up the chimney. We could imagine a surface surrounding a 'source', count the number of stream lines coming out of it and subtract any going into it. The result measures the

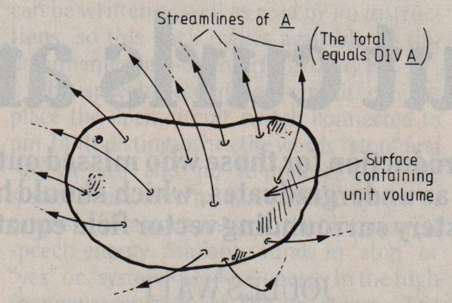

Fig.3. The excess of vector field lines issuing out of unit volume at a point in a field over those going in, is the measure of divergence.

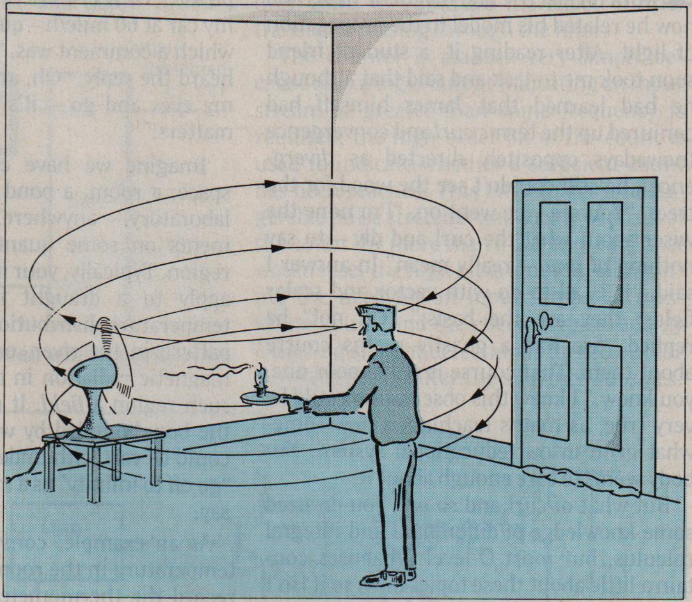

Fig.4. Completely closed stream lines still means a vector field – but one in which there are no sources or sinks.

strength of the source of the vector. The generating source 'material' is a scalar. It might be concentrated at definite locations or distributed around the region as a scalar field. Measuring the vector field lines generated in this way coming from unit volume at any point is called finding the *divergence* of the vector field, and it is written

$$\mathrm{div}\mathbf{D} = \rho \text{ or } \nabla . \mathbf{D} = \rho$$

where **D** is the vector and ρ is the source scalar quantity. ∇ is the differential vector operator again. ∇**.D** measures how rapidly **D** is appearing per unit volume at a point, in other words how concentrated the generating source quantity is there.

VECTOR FIELDS: CURL

Vectors can possess a different property. A second vector field can be derived from the first one. Imagine we are in the draughty room again. All the keyholes, fireplaces, door surrounds might be well and truly blocked, yet here is a huge draught roaring past our ears. Then we notice the large fan whirling the air round and round. We soon realise that the flow lines form complete closed paths or loops, like those in Fig.4. They do not start and stop on any sources. Further investigations show that there are little closed flow loops distributed all over the points of the region. At some points the circulation whirls vigorously, at others rather weakly – or hardly at all. These

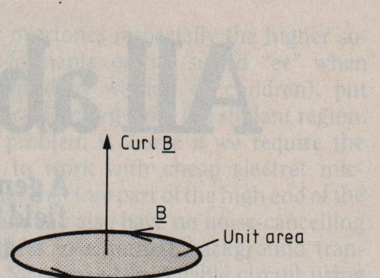

Fig.5. The measure (per unit area) of the circulation of the field lines at a point is the curl of the vector.

vortices rotate in planes whose orientations vary from point to point.

We can draw an imaginary line through every little plane loop according to the strength of the vortex per unit area at each location. The lines point away at right angles to the circulation planes in such directions that the rotations go round them in the sense of a corkscrew, as Fig.5 shows. Joining all these new lines point by point gives another vector field. This is the *curl* of the first one. It measures the vorticity per unit area:

$$\mathrm{curl}\mathbf{B}=\mathbf{A} \text{ or } \nabla\times\mathbf{B}=\mathbf{A}.$$

You might notice the significant result about the curl airising from this discussion. **A** and **B** are always at right angles everywhere.

The divergence operation gives a scalar field from a vector, whereas the curl gives another vector field from a vector. The general vector field consists of combinations of these two extremes. Separating the general field into its 'curl' part and its 'div' part is called *Helmholtz's Theorem*.

The main point I make now is that the equivalent to scalar products in the vector *field* situation is the div operation, while the curl is a vector product operation. The meanings of div and curl might now appear a little less daunting.

You might have already noticed that as curl fields start and end on themselves, you can never have a divergence of such a field.

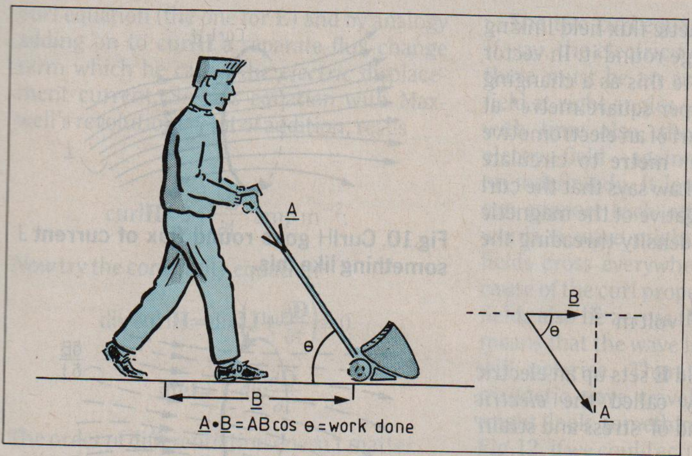

Fig.6. Work done by the component of a force shows a typical example of the scalar product of two vectors.

We can write this as

$$\text{div curl}\mathbf{A} \equiv 0.$$

Again, div fields, and grad of scalar fields never have lines that loop round and end on themselves. This mean that

$$\text{curl div}\mathbf{D} \equiv 0$$

or $$\text{curl grad}\varphi \equiv 0$$

always.

VECTOR DIFFERENTIAL OPERATOR

The vector differential operator ∇, is admittedly hard to visualize. It obeys the vector rules for products etc., but also it is as we have seen, a differential operator and works on variable and functions 'to the right'. In other words, $\nabla.\mathbf{A}$ is not the same as $\mathbf{A}.\nabla$, where in the second expression ∇ is looking for something to operate on to the right of it. ∇ might be asked to operate on (that is, differentiate) a product – such as $\nabla.(\mathbf{A}\times\mathbf{B})$ or $\nabla\times(\mathbf{A}\times\mathbf{B})$. The rule for differentiating a product has to be obeyed *and* the rules for scalar and vector product expansion at the same time. For example[4]

$$\text{curl curl}\mathbf{A} = \text{grad div}\mathbf{A} - \nabla^2\mathbf{A}$$

where ∇^2 is a second-order or double differentiation. Or again

$$\text{div}(\mathbf{E}\times\mathbf{H}) = \mathbf{H}.\text{curl}\mathbf{E} - \mathbf{E}.\text{curl}\mathbf{H}.$$

Both these expansions give valuable and concise results and descriptions in the mathematical modelling of the electric and magnetic vector field distributions that occur in e.m. theory.

CONCISE NOTATION

The value of vector notation lies in its conciseness. A number of results, like the above two, give a succinct view of what is going on. Nevertheless, we usually return to old René Descartes 'scaffolding' – the x, y and z axes, when doing real life problems. This applies especially to engineering situations with rectangular symmetry, for example rectangular waveguides. If the waveguide is a circular one, the coordinates might be the cylindrical set r, θ, z. If an aerial is radiating into a sphere, we might choose the spherical coordinates r, θ, φ.

All of these coordinate systems have axes which are at right angles to each other at any

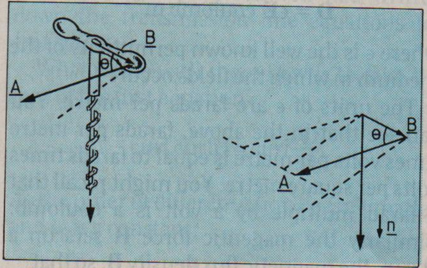

Fig.7. The turning force at the end of an arm – like that on the corkscrew shown, constitutes an example of the vector product.

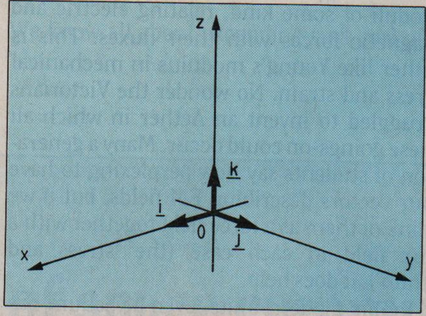

Fig.8. The cartesian axes form the most common 'scaffolding' upon which we erect the dimensions and boundaries of real technical and engineering problems. When we want to show the vector properties in particular, the unit vectors i, j and k are used to point the way.

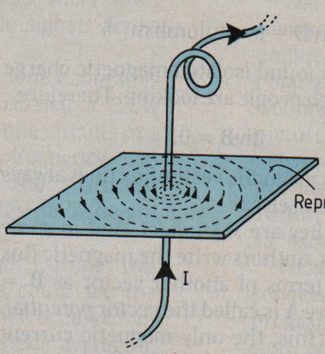

Fig.9. Although the iron filings in this famous old experiment are not curlH, nevertheless they lie along the lines of circulation of the magnetic field. Then if you imagine the amount of this circulation per unit area in the region, then you have the idea of curlH there.

Vectors

Running away into the above discussion, I have put the cart before the horse, at least by assuming everybody knows about ordinary vectors and how to multiply them. Quite a number of people probably don't.

Adding two vectors is simple. We use the 'parallelogram' or 'triangle' rule that most people no doubt did learn. Products of vectors give rise to a little more thought. The results all depend on how we define the multiplication and this can be done in a number of ways. Over the years, it has all boiled down to two different products – one producing a scalar result, the other giving a new vector. These products have survived, indeed have developed into 'standards' because they fit well into how physics, engineering etc. describes things.

The first kind, Fig.6, known as the *scalar product* – or 'dot' product as some people call it, is simply

$$A.B = \mathbf{AB}\cos\theta$$

where A and B are the two vectors with magnitudes A and B and an angle θ between them. This mens that if the vectors are at right angles there is no product, because cos90° is zero. Of course, this product is the multiplication of one magnitude by the magnitude of the *resolved component* of the other vector along the direction of the first. For example, also in Fig.6, it gives the work done by a force (A, say) moving along B, (The direction of A in general being at angle θ to B). However, the greatest value of A.B occurs when the vectors are parallel.

The other product, illustrated in Fig.7, we call the *vector* or 'cross' product. It is

$$A\times B = \mathbf{NAB}\sin\theta$$

Now you see that the resolution takes the other component, the 'sine' one and the result is greatest for normal, or 'orthogonal' vectors this time. N is the *direction vector* or *unit vector* such that if A is turned towards B, N goes in the direction of an ordinary corkscrew.

If we consider B×A instead, then N points the other way. This means A×B is *not* equal to B×A. In other words, A and B do not commute in the cross product. Often this is the first time students come across a noncommutative algebra, (and some of them come to grief at first). In fact,

$$A\times B = -B\times A$$

At this stage in proceedings, if you look into a book dealing with vectors[4], you can have an entertaining time working out some of the multiple products like

$$A\times(B\times C)$$

or $$A.(B\times C)$$

Nobody has give a meaning to *division* by a vector, so you won't come across that operation. This means that the denominator of differentiations always contain scalars, or scalar components of a vector, and so on.

It is normal practice to typecast vectors in bold face or, as an alternative to underline the characters, as in our drawings. Here they are in light face as the text is in bold.

point in space: in other words they form orthogonal sets. Admittedly, as Fig.8 might remind you, the rectangular set x, y, z is the simplest.

If we put the properties of scalar and vector fields into cartesian coordinates, we get the three sets of equations that describe the three components of each field.

∇ is equivalent to $\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}$

in cartesians. This shows the three components explicitly. **i**, **j** and **k** are the unit vectors along the x, y, and z axes.

Therefore gradφ, which is $\nabla\varphi$, has the three components

$$\mathbf{i}\frac{\partial\phi}{\partial x}+\mathbf{j}\frac{\partial\phi}{\partial y}+\mathbf{k}\frac{\partial\phi}{\partial z}$$

in cartesians and is a vector.

div**A**, also written $\nabla.\mathbf{A}$, is

$$\left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}\right).(\mathrm{i}A_x+\mathrm{j}A_y+\mathrm{k}A_z)$$

which, when we notice **i.i**=1 and **i.j**=0 etc., is

$$\frac{\partial Ax}{\partial x}+\frac{\partial Ay}{\partial y}+\frac{\partial Az}{\partial z}.$$

curl**B**, alternatively $\nabla\times\mathbf{B}$, is

$$\left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}\right)\times(\mathrm{i}B_x+\mathrm{j}B_y+\mathbf{k}B_z)$$

Keep a sharp eye on the definition of the vector product and you will see that $\mathbf{i}\times\mathbf{i}$ etc $=0$, but $\mathbf{i}\times\mathbf{j}=\mathbf{k}$, $\mathbf{i}\times\mathbf{k}=-\mathbf{j}$ and so on, which yields

$$\mathbf{i}\left(\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}\right)+\mathbf{j}\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)+\mathbf{k}\left(\frac{\partial B_x}{\partial x}-\frac{\partial B_z}{\partial y}\right)$$

for the three components of the vector curl**B**.

The curl expansion, definitely the most complicated, has a determinant expression so that you can remember it easily

$$\mathrm{curl}\mathbf{B}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ B_x & B_y & B_z\end{vmatrix}$$

MAXWELL'S EQUATIONS AND E M WAVES

With our little nibble at vector fields, we arrive at one of the most famous sets of equations to grace the table of science and engineering. Öersted showed that a magnetic field **H** amps per metre sprang up round a current flow in a conductor. You may remember the science master demonstrating this with a card and iron filings, Fig.9.

Ampère wrote down a mathematical statement about what might be going on. Using our modern notation and units, together with the idea of the current flow paths as a vector field distribution, like that in Fig.10, Ampère's description amounts to saying that the curl of the magnetic force at any point is equal to the current density **J** streaming through

$$\mathrm{curl}\mathbf{H}=\mathbf{J}\text{ amps m}^{-2}.$$

Michael Faraday found the inverse effect, namely his Law of Electromagnetic Induction. A *changing* magnetic flux field linking a circuit causes a voltage round it. In vector field terms we can write this as a changing flux density **B** webers per square metre* at any point sets up the curl of an electromotive force field **E** volts per metre to circulate round it. So Faraday's Law says that the curl of **E** is equal to the negative of the magnetic displacement current density threading the region, see Fig.11

$$\mathrm{curl}\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\text{ volt m}^{-2}$$

An electric force field **E** sets up an electric flux field **D**, usually called the *electric displacement*, in a kind of 'stress and strain' relationship.

$$\mathbf{D}=\epsilon\mathbf{E}\text{ coulomb m}^{-2}$$

where ϵ is the well known permittivity of the medium in which the fields occur.

The units of ϵ are farads per metre. You can see that in the above, farads per metre times volts per metre is equal to farads times volts per square metre. You might recall that a farad multiple by a volt is a coulomb. Similarly the magentic force **H** sets up a strain, the magnetic flux density **B**, so that

$$\mathbf{B}=\mu\mathbf{H}\text{ weber m}^{-2}\text{ or tesla}$$

where μ is the permeability in henrys per metre.

The ϵ and μ look like 'stress and strain' moduli of some kind, relating electric and magnetic forces with their fluxes. This is rather like Young's modulus in mechanical stress and strain. No wonder the Victorians struggled to invent an Aether in which all these goings-on could occur. Many a generation of students say how perplexing to have four vectors describing EM fields, but if we think of them as a force field together with a flux field in each case (the 'stress and strain'), it does help.

In our modern units, even a vacuum has values for ϵ and μ. They are

$$\epsilon_0=\frac{1}{36\pi\times10^9}\text{ farad m}^{-1}$$

$$\mu_0=4\pi\times10^7\text{ henry m}^{-1}$$

Electric charge gives rise to the flux field **D**. If there is a concentration ρ coulombs per unit volume distributed in a region, then

$$\mathrm{div}\mathbf{D}=\rho\text{ coulomb m}^{-3}.$$

Nobody has found isolated magnetic charge yet, although people are looking. Therefore

$$\mathrm{div}\mathbf{B}=0.$$

This means magnetic vector flux lines always close on themselves.

We say they are *solenoidal*. And in some discussions, authors write the magnetic flux density in terms of another vector as $\mathbf{B}=\mathrm{curl}\mathbf{A}$, where **A** is called the *vector potential*. Because of this, the only magnetic current that can flow is magnetic displacement current – as we saw in Faraday's Law.

We have one last result, not usually included in Maxwell's equations, which relates the current flow lines coming out of a region to the rate of change of charge per unit volume at each point in it. This is the 'equation of continuity'

$$\mathrm{div}\mathbf{J}=-\frac{\partial\rho}{\partial t}\text{ coulomb m}^{-3}\text{s}^{-1}\text{ (or amp m}^{-3}\text{)}$$

Now here is an interesting conundrum, a problem Maxwell solved although not quite this way. From your knowledge of vectors, and the equation of continuity, you might try getting the continuity result from the curl**H** equation by taking its divergence

$$\mathrm{div\ curl}\mathbf{H}=\mathrm{div}\mathbf{J}=-\frac{\partial\rho}{\partial t}$$

This looks alright – but hold on, div of a curl is *always* zero. Apparently we can never have a charge changing with time! Maxwell got out of this difficulty by looking at the other

*The S.I. Committee asks us to call a weber m^{-2}, a *tesla*.

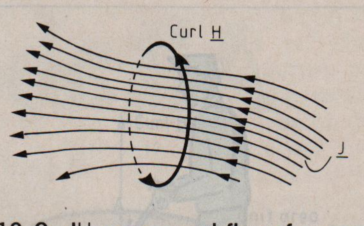

Fig.10. CurlH goes round flux of current J something like this.

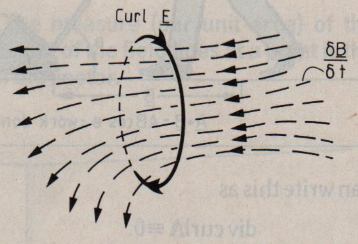

Fig.11. Similarly for the rate of change of flux density B. But notice the direction of curlE in this case.

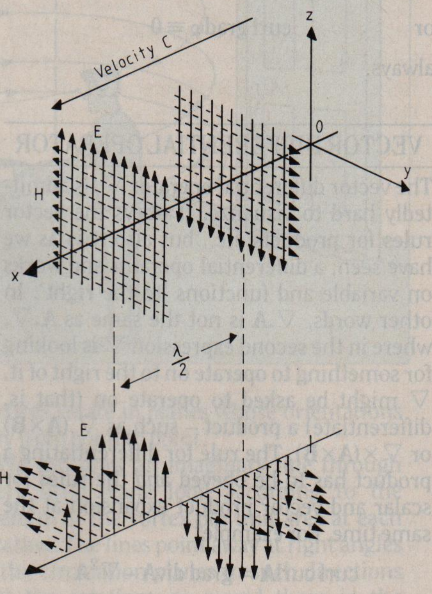

Fig.12. The EM wave advances somewhat like this diagram suggests. The E and H field lines cross at right angles to each other and with the direction of travel. For a single frequency, the lines distribute along the propagation axis in strength and with reversal's depicted in the lower part of the diagram.

curl equation (the one for **E**) and by analogy adding on to curl**H** a separate flux change term which he called the *electric* displacement current. So this equation with Maxwell's revolutionary bit of addition, reads

$$\text{curl}\mathbf{H}=\mathbf{J}+\frac{\partial \mathbf{D}}{\partial t}\ \text{amp m}^{-2}.$$

Now try the continuity equation

$$\text{div curl}\mathbf{H}=\text{div}\left(\mathbf{J}+\frac{\partial \mathbf{D}}{\partial t}\right)\equiv 0$$

$$\therefore\ \text{div}\mathbf{J}=-\text{div}\frac{\partial \mathbf{D}}{\partial t}$$

The order of differentiation doesn't matter

$$\therefore\ \text{div}\mathbf{J}=-\frac{\partial}{\partial t}\text{div}\,\mathbf{D}$$

and from the fact that div**D**$=-\rho$ we have directly

$$\text{div}\mathbf{J}=-\frac{\partial \rho}{\partial t}$$

So at last we have Maxwell's equations. For example, out in space, away from currents **J** and charges ρ, the equations are

$$\begin{aligned}&\text{curl}\mathbf{H}=\frac{\partial \mathbf{D}}{\partial t}\\ &\text{curl}\mathbf{E}=-\frac{\partial \mathbf{B}}{\partial t}\quad \text{with}\ \left\{\begin{array}{l}\mathbf{D}=\epsilon_0\mathbf{E}\\ \mathbf{B}=\mu_0\mathbf{H}\end{array}\right.\\ &\text{div}\mathbf{D}=0\\ &\text{div}\mathbf{B}=0\end{aligned}$$

The two curl equations tell quite a story. If, say, the electric field changes with time, there must be an accompanying magnetic field at right angles to it. If this is changing with time also, then it sets up another electric field – again at right angles. And so on indefinitely. It looks as though **E** and **H** can support each other in space. In other words, a *wave* might exist. Notice the two fields cross everywhere at right angles because of the curl property relating them. The fields also lie across the line of travel, which means that the wave is a transverse type and will polarize. Therefore a plane electromagnetic wave travelling along the x axis might look something like that shown in Fig.12, if we could actually see the lines. This set of results shows Maxwell had written down the transmission line equations for free space.

What we do with them now is to take the curl of the first equation

$$\text{curl curl}\mathbf{H}=\text{curl}\frac{\partial \mathbf{D}}{\partial t}.$$

As the order of differentiation doesn't matter and ϵ_0 is a constant,

$$\text{curl curl}\,\mathbf{H}=\frac{\partial}{\partial t}\text{curl}\mathbf{D}=\epsilon_0\frac{\partial}{\partial t}\text{curl}\mathbf{E}.$$

But from the second curl equation, Curl $\mathbf{E}=-\partial\mathbf{B}/\partial t$, so insert it

$$\text{curl curl}\mathbf{H}=-\mu_0\epsilon_0\frac{\partial \mathbf{H}}{\partial t^2}.$$

We know all about expanding curl curl of a vector from our previous discussions, therefore the *piéce de résistance*:

$$\text{curl curl}\mathbf{H}=\text{grad div}\mathbf{H}-\nabla^2\mathbf{H}=\mu_0\epsilon_0\frac{\partial \mathbf{H}}{\partial t^2}$$

$$\therefore\ \nabla^2\mathbf{H}=\mu_0\epsilon_0\frac{\partial \mathbf{H}}{\partial t^2}.$$

This is the famous wave equation. Here it describes the magnetic field part of the electromagnetic waves Maxwell predicted. The electric field has a similar wave equation. The multiplier of the right hand, or 'time' term is always in this type of equation, which, by the way, is called d'Alembert's equation, where c is the velocity of the wave.

This means that for our derivation with Maxwell's equations. And what is more, c works out to be very nearly 300 000 000 metres per second from the measured values of ϵ_0 and μ_0. This is the velocity of light in a vacuum.

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4. Ref. 3. is pithy and interesting. For a good modern treatment, see: "Field and Wave Electromagnetics", Chap. 2. by D.K. Cheng. Addison-Wesley, (1983).

25 years of space communications

A quarter of a century ago this month the era of international communications through space began. Telstar, the world's first commercial communications satellite, was launched by NASA into low earth orbit on 10 July, 1962, and within days it had successfully relayed television, telephone and facsimile signals across the Atlantic in both directions.

Built in the United States by Bell Telephone Laboratories, Telstar weighed just 77kg and it carried a 2.5W transmitter capable of handling 600 telephone circuits or a single television channel; the uplink frequency was 6.39GHz and the downlink 4.17GHz. The spacecraft orbited the earth once every 156 minutes, at a height varying between 910 and 5875km.

Communication between the US and Europe was possible only when both ground-stations could see the satellite: about three or four useful orbits occurred in every 24 hours, offering transmission periods averaging about 30 minutes. And it was to maximize these that the General Post Office (British Telecom's predecessor) erected its dish close to the western extremity of the UK mainland, at Goonhilly Down.

Geostationary satellites, which would later allow transmission for extended periods, were to come much later – though their principle was by then well known, having first appeared in print in *Wireless World* in February 1945, in a letter from science writer Arthur C. Clarke.

The first night's tv transmissions from Telstar were marred by poor reception: through a misunderstanding, engineers on either side had adopted opposite planes of polarization. But by the following evening, staff at Goonhilly had made the necessary adjustments and live pictures were exchanged between the BBC and US networks. Colour pictures from a 525-line slide scanner provided by the BBC Research Department were sent on 16 July; The US station at Andover, Maine, reported them as "excellent".

Today there are some 130 communications satellites in service, handling traffic between 160 countries. Satellite programme feeds are routine events on radio and television, and the era of direct satellite broadcasting into the home is almost with us. Goonhilly now has ten aerials working to satellites over the Atlantic and Indian oceans; the first of them, the dish used for Telstar, is still operational.