Maxwell's e.m. theory revisited

If Maxwell's theory is about to be displaced according to the many words in this journal recently, we might take a nostalgic look at it once again.

'JOULES WATT'

"Back to basics," we said, "before expecting a profound paradigm shift. You ought to know a little about the accepted norm." If Maxwell's theory is about to be displaced (no pun!) according to the many words in this journal recently, we might take a nostalgic look at it once again.

"You don't mean, er, those – *curls* and things...?" Not directly, but the curls – and the grads, divs, dels – do seem to remain unpopular with students, probably the reason is bad teaching again...

Yet the developed theory of electromagnetism still holds sway. If there are some phenomena such a theory does not explain, then any new model must explain all that has gone before – plus the new aspects. At least that is the way Thomas Kuhn[1] outlined the situation..

But Maxwell remains a good model, displacement current and all. In saying this, I have mentioned a vital point. It *is* only a model, a kind of template held up against nature, as it were. If the picture fits, so well and good, we can predict some occurances and design a few things and earn some money.

Is it *true*? That is not a relevant question. We don't really care if it is absolutely true, we can never know that anyway. The point is, does it work? If yes, we go ahead and make our name, or even earn the money... Science and technology is pragmatic, whether pragmatism (in William James's sense[2]) is out of fashion or not.

For example, electrons – are they *really* there? Is displacement current real? A number of people have become hot under the collar recently (and not so recently), about the 'truth' of these ideas. But they have missed the point. Nothing is ever with certainty *proved* in science and therefore neither is the engineering based on it. It can only be refuted, when it fails to produce the goods. This time, Karl Popper[3] had a few words to say on the subject, albeit my limited comments are a only a brief scratch on the surface.

WHAT DID MAXWELL SAY AND WHY?

When James Clerk Maxwell was at Edinburgh University, he came into contact with the philosopher William Hamilton. In the cut and thrust of ideas, the relativity of human knowledge held sway, because Hamilton taught that we only know relations between things – not much about things in themselves. This links back to a Kantian view. All this affected the young Maxwell deeply.

At the same time, Maxwell came up against the strict teaching and acute experimental methods of the physicist James Forbes, which also impressed him. It left Maxwell always aware that his theoretical constructs must be refinable in the fires of experimental verification – a view rare in theoretical physicists.

LINES OF FORCE

A little later, Maxwell deeply appreciated the work of Michael Faraday and one of his first important papers[4] on Electromagnetism was his "On Faraday's Lines of Force" (1856). The 'mechanical' properties of the imaginary lines included the tension in length (which explained attraction) and their repulsion sideways (explaining repulsion itself). Maxwell modelled these properties mathematically.

This first paper was followed in 1861 by the paper "On Physical Lines of Force"[5], because in the meantime William Thomson, later Lord Kelvin, had been in lively correspondence with Maxwell, and between the pair of them they had noticed all the analogies between: stream lines in fluid flow, lines of heat flow, electric current flow lines, lines of force in electric fields and lines of force in magnetic fields.

These analogy relationships give partial explanations. They are models, but cross fertilise thinking about different branches of physical science. Yet they warn us not to think electricity is really a fluid water, or really like a state of heat flow agitation...

Further discussion, this time with Stokes, had Maxwell contemplating Stokes' work showing that heat flow in a non-uniform crystal had a direction **A** not always parallel to the direction of maximum temperature gradients θ.

$$\mathbf{A} = \mathbf{T}\theta$$

where **T** is a tensor, describing the anisotropic crystal. Maxwell immediately applied the analogy to magnetism and distinguished the two vectors which he called the "flow" **B** and the "force" **H** and realised that in an anisotropic magnetic medium (like some of our modern ferrites), the lines of force would not always be parallel to the lines of flux. The direct analogy in the electric case was flow lines of current density **J**, with the force **E** setting them up.

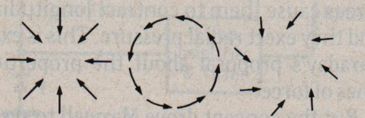
The trouble is that generations of students have been perplexed by these two 'different' vectors **B** and **H** describing the same thing – magnetic field. You might have found this because of bad teaching again and a glance at these original papers often helps.

VECTORS

The analogies led Maxwell to discourse on two classes of vector functions existing in general, *fluxes* and *forces*. A flux **B** is subject to a continuity equation and is integrated over a surface. The picture is 'streaming across'. A force **P** is a vector which is usually derived – but not always – from a single valued scalar function, the *potential* and is integrated along a line. It gives the concept 'force along'. The vectors **B** and **J** are fluxes, **H** and **E** are forces.

In Maxwell's earlier discussions and growing mental pictures, he stuck to three-dimensional Cartesians (the x,y, z axes). But by 1870 after much correspondence with Peter Guthrie Tait and William Thomson, Maxwell[6] himself advanced the ideas of *convergence* (negative divergence), the *curl* and *slope* (later called the gradient). The extract from his paper is interesting:

> ". . . $\nabla\sigma$ has, in general, also a vector portion, and I propose, but with great diffidence, to call this vector the *Curl* or *Version* of the original vector function.
>
> It represents the direction and magnitude of the rotation of the subject matter carried by the vector σ. I have sought for a word which shall neither, like Rotation, Whirl, or Twirl, connote motion, nor, like Twist, indicate a helical or screw structure which is not of the nature of a vector at all."



Convergence Curl Convergence and curl

Maxwell found Tait's enthusiasm for 'quaternions' invented by William Hamilton (not the philosopher, but another Hamilton, the mathematician), had given him the germ of vector analysis – especially via the use of Hamilton's operator ∇. There was much humour in Maxwell's correspondence about ∇ and his play on words regarding the possible names for it; Nabla, or even Atled had been suggested[7].

Maxwell did not fully adopt the complicated quarternion ideas, but used the form,

$$\nabla = \mathbf{i}\frac{d}{dx} + \mathbf{j}\frac{d}{dx} + \mathbf{k}\frac{d}{dx}$$

and realised the vector properties of it in connection with the 'div' and 'curl' operations. It remained to Willard Gibbs in a pamphlet and Oliver Heaviside[8] to oust 'quaternions' but to bring in the full modern vector analysis notation. You will still find it most entertaining to read Heaviside's pithy comments about Clarendon type faces and other notations. Maxwell and certainly Heaviside would immediately recognise our modern presentation of the equations.

Advancing an argument started by Thomson, Maxwell showed that any flux vector had two parts. It had a component from the curl of a force vector plus another part which was the gradient of a scalar function. For magnetism he wrote,

$$\mathbf{B} = \text{curl}\mathbf{A} + \text{grad}\Psi$$

and went on to say that in the absence of magnetic poles (or isolated magnetic charge) there are no sources and therefore $\text{grad}\Psi = 0$.

Therefore he had obtained a complete set of equations between **B**, **H**, **J** and **E**. At this stage, still using Cartesian mathematical arguments, Maxwell showed Faraday's electromagnetic induction is described in our modern notation by,

$$\text{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

and from this, by using **B** = curl **A** showed

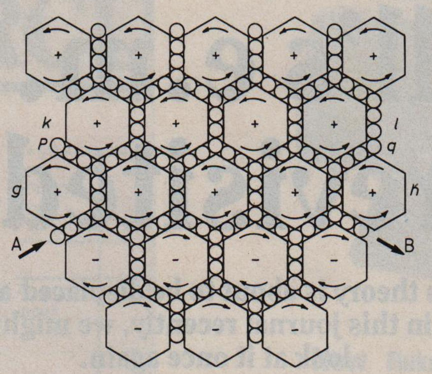
$$\text{curl}\mathbf{E} = \text{curl}\left(\frac{\partial \mathbf{A}}{\partial t}\right) \text{ or } \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}.$$

Maxwell called this new function **A** the "electrotonic state" in recognition of Faraday's speculations about a hypothetical state of stress that must surround electrically or magnetically 'charged' bodies.

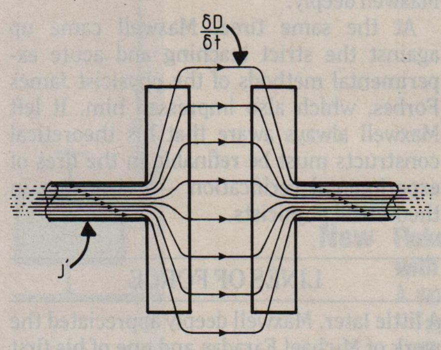
STRUGGLES WITH MECHANICAL ANALOGIES

You will find Maxwell's struggles with how the fields extend around the sources contained in the second "lines of force" paper[5]. He tries analogy again with a kind of mechanical vortex model, see Fig.1. He extended the model from matter to space, postulating an *ether* to contain the vortices. Consider the array of vortices embedded in an incompressible fluid. When they rotate, centrifugal forces cause them to contract longitudinally and they exert radial pressure. This is exactly Faraday's proposal about the properties of lines of force.

But the concept drove Maxwell to see that electricity was not confined to a fluid in the



Maxwell's Figure 2 in his paper, "On Physical Lines of Force". The electric current was represented by the 'ball bearings' running from A to B and the resulting vortex motion was given to the imaginary 'cells' in the surrounding space as shown. The line p to q shows what would happen if another conductor was placed along there, thus explaining induction. One or two of the rotation direction arrows are incorrect.



The flow lines of a current form closed paths according to Maxwell. This means they must pass through the dielectric of a capacitor, including a vacuum. All the current there must be in the form of Displacement or Electric Flux variations as no actual electrons are emitted through the region.

conductor on this view of things, but was disseminated in space – and the energy was 'stored' in the space containing the fields... The function **A**, which we now call the vector potential, acted as a kind of momentum term in the field. The equation $\mathbf{E} = \partial\mathbf{A}/\partial t$ was equivalent to Newton's equation between force and rate of change of momentum.

Now Maxwell hit upon the idea that the medium containing the vortices was elastic – hence the energy storage in the medium was by an elastic distortion. Two remarkable consequences quickly follow. Since the space surrounding a conductor is capable of an elastic displacement – a varying field displaces an equivalent current. This is the first glimmering of the "displacement current" postulate. Secondly, any elastic medium with density ρ and a shear modulus m can transmit transverse waves with a velocity,

$$v = \sqrt{\frac{m}{\rho}}.$$

Maxwell inserted magnetic and electric quantities (based as we now say on permittivity and permeability) and found the wave velocity would almost equal the then accepted value of the velocity of light. With some excitement he wrote in the "Lines of Force..." paper

> "The velocity of light in air, as determined by M. Fizeau*, is 70,843 leagues per second (25 leagues to a degree) which gives
>
> V=314,858,000,000 millimetres
> = 195,647 miles per second........(137)
>
> The velocity of transverse undulations in our hypothetical medium, calculated from the electro-magnetic experiments of MM. Kohlrausch and Weber, agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau, that we can scarcely avoid the inference that *light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena"*.

By 1865 Maxwell had written his paper "A Dynamic Theory Of The Electromagnetic Field"[9]. In it, he spelt out the full development of how the electromagnetic waves would propagate. Note the word "field" appears for the first time in the title. He had dropped the "vortices" intermediate analogy stage and relied on a few facts including the really original concept of the 'displacement current'. He effectively noted that the magnetic current is always a 'displacement current' $\partial\mathbf{B}/\partial t$ as there is nomagnetic charge in the Universe. Therefore why not *some* of the electric current at least in the form $\partial\mathbf{D}/\partial t$? The total current then, is always closed and is a set of flow lines

$$\mathbf{J}' = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

where **J**' is the total current, **J** the conduction current and $\partial\mathbf{D}/\partial t$ is the displacement current, **D** now being the *electric flux* or *displacement* vector. A changing current might set up a flow pattern in a capacitor like that in Fig.2.

ELECTRICITY, MAGNETISM AND LIGHT

Maxwell saw the significance of his construct. He wrote to his cousin, Charles Cay, "I have also a paper afloat, containing an electromagnetic theory of light, which till I am convinced to the contrary, I hold to be great guns."

Again, the philosopher Hamilton's influence on Maxwell in his youth can be seen. The build up via analogies, his development of the mechanical model – and then dropping it, and finally the analysis of the direct relations between the two classes of phenomena (magnetism and electricity) as a unifying structure – are all based on Hamilton's doctrine of the relativity of knowledge. Einstein said of Maxwell that he saw the future role of field theory in physics, complete with its describing differential equations and seeing that was his stroke of genius.

The revolutionary idea is not really the displacement current proper, (in spite of the heat under many collars!), but the whole 'dissemination' idea into the medium. Maxwell's formal energy densities in the medium which also link with some of Thomson's work, encapsulate this view:

magnetic energy density = **B.H** } joule m^{-3}
and electric energy density = **D.E** }

Maxwell writing at the end of his "Dynamics..." paper, even calculated the peak value of the electric field in sunlight, both in the solar constant at the Earth's surface and at the sun.

"The energy passing through a unit of area is

$$W=\frac{P^2}{8\pi\mu V};$$

so that $P=\sqrt{8\pi\mu VW}$,

where V is the velocity of light, and W is the energy communicated to unit of area by the light in a second.

According to Pouillet's data, as calculated by Professor W. Thomson, the mechanical value of direct sunlight at the Earth is 83.4 foot-pounds per second per square foot. This gives the maximum value of P in direct sunlight at the Earth's distance from the Sun, P=60,000,000, or about 600 Daniell's cells per metre. At the Sun's surface the value of P would be about 13,000 Daniell's cells per metre."

The model Maxwell gave explained and predicted optical and electrical phenomena with great vigour and precision. Whichever vectors you take, strictly transverse waves in space appear because of the vector product nature of curl. The equations are the 'telegraphers equations' of space, and look like the transmission line equations Heaviside derived for waves on wires later:

$$\mathrm{curl}\,\mathbf{H} = J + \frac{\partial D}{\partial t} \quad \text{with} \quad D = \epsilon E$$

$$\mathrm{curl}\,\mathbf{E} = -\frac{\partial B}{\partial t} \qquad B = \mu H$$

$$J = \sigma E$$

$$\mathrm{div}\,\mathbf{B} = 0$$

$$\mathrm{div}\,\mathbf{D} = \rho.$$

The wholly transverse solutions eliminated the longitudinal wave requirements that had embarrassed earlier theories of light.

Heinrich Hertz's electrical generation and detection of long maxwellian waves, with all the predicted properties was a supreme vindication. The vast proliferation of engineering uses of these waves up to the present time is a further supreme example.

PREDICTING POWER AND USEFULNESS BUILD UP

But just as important, Maxwell predicted that the waves would exert a *radiation pressure* – thus disposing of the idea that any luminiferous pressure would be a crucial argument for a corpuscular theory of light. Lebedev proved the radiation pressure postulate experimentally in 1900. It explains the repulsion of parts of the tails of comets. Such radiation pressure is vital for Black Body radiation theory. It may be used to derive classically the time dilation formula of special relativity and explains how stars hold up internally, together with their allowed mass range...

Maxwell's famous Treatise[10] sets out the on-going work in book form, but adds little more to his papers and memoirs. In the late 1870s he was about to write a deeper investigation into all these researches, but stomach cancer heralded his early death aged 48 in November 1879 – at the prime of his powers. As usual, we always speculate on what he might have achieved had he lived.

References

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3. K.R. Popper, Conjectures and Refutations R.K.P. 1972.
4. J. Clerk Maxwell, On Faraday's Lines of Force, Scientific Papers, 1855, 1856, reprinted by Dover, New York, 1952.
5. J.Clerk Maxwell, On Physical Lines of Force Scientific Papers, 1861, 1862.
6. J.Clerk Maxwell, On the Mathematical Classification of Physical Quantities, Scientific Papers.
7. Maxwell had much to say with Tait, Thomson, and many others via the 'halfpenny post' after the Post Office introduced it in 1869. Tait was known as T' and Thomson as T. These two authors wrote a "Treatise on Natural Philosophy" which was reviewed by Maxwell – who henceforth referred to it as "T and T'". Tait couldn't stand Tyndall, another scientist in the milieu, and referred to him as T" ('where T" is a quantity of the second order...'). Tait had written a book on Thermodynamics in which he had given an equation dp/dt=JCM. Maxwell then signed his cards:"...yours sincerely, dp/dt."
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10. J.Clerk Maxwell, Treatise on Electricity and Magnetism, Oxford, 1873, 2nd. ed. 1881, 3rd. ed. 1891.

Dividers faster than 3GHz

Bipolar devices offer better phase noise and speed power performance than gallium arsenide equivalents

Advances in semiconductor processing and photolithography at Plessey have produced the first prescaler i.cs for frequencies above 3GHz. They incorporate silicon bipoloar transistors with 1.5μm emitters and 7GHz f_t at 0.5mW dissipation. Coupling these transistors with 5μm-pitch metal combines the high speed with high packing density.

Work is currently being done to increase speed of the new dividers to 6GHz, which will make them useful for applications like directly synthesizing local oscillators in C-band satellite receiver front ends.

Initially, the SP8800 prescaler series consists of divide-by-two, four, eight and ten i.cs in surface-mount and dil packages. Sensitivity and overload performances are good, as Fig. 1 shows, and power dissipation/radiation are low. Being bipolar, the devices inherently offer better phase-noise and speed/power performance than GaAs equivalents.

Applications include counter prescaling and frequency synthesis. Figure 2 represents a 3.5GHz frequency synthesis loop with one of the new prescalers dividing by four and an SP8704 dividing by 128 or 129. With the world's first military-specification 20mA 1.5GHz synthesizer, Fig. 3, it will be possible to produce a two-chip military-grade frequency synthesis loop.

The SP8850 is currently under development; samples should be available in October.

Nick Cowley

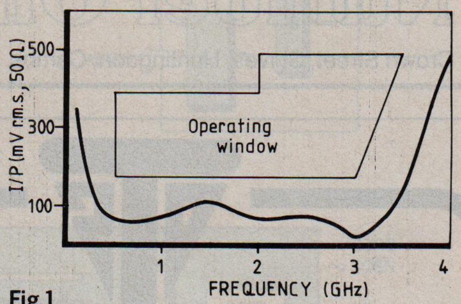

Fig.1

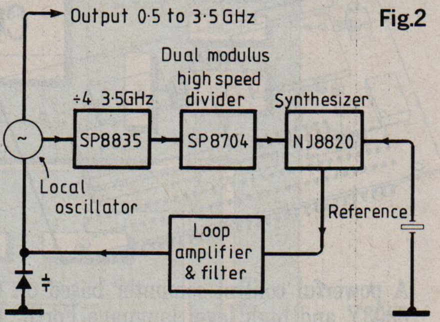

Fig.2

Fig.3